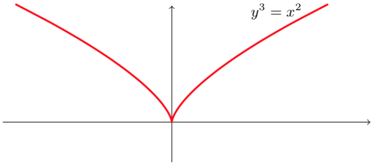
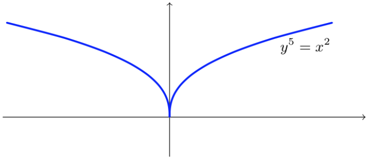
Why don't we see C^2-singularities?

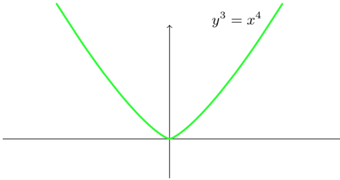
What we see – and what we don't see

Let us start with three real plane algebraic curves

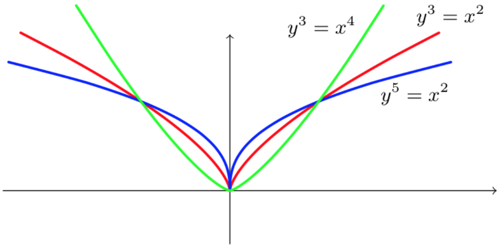
$$x^2 = y^3 \qquad\qquad x^2 = y^5 \qquad\qquad x^4 = y^3.$$

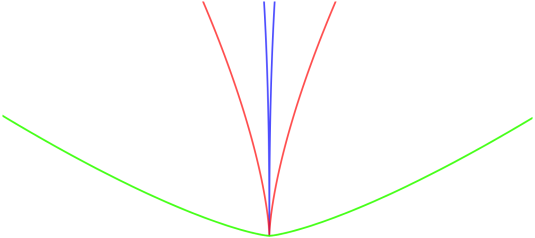






(drawing courtesy of Hana Melánová)


$y^3 = x^4$
$y^3 = x^2$
$y^5 = x^2$



Curvatures

Curvatures $\qquad t \to 0$:

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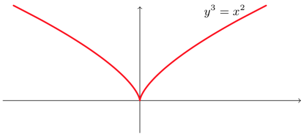
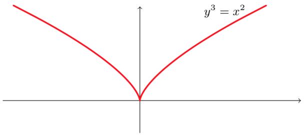


$$t \to (t^3, t^2) : \kappa = t^2/t^3 \longrightarrow \infty$$

Curvatures $\quad t \to 0$:



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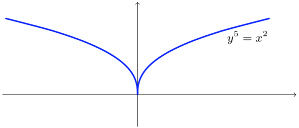

$$t \to (t^5, t^2) : \kappa = t^4/t^3 \longrightarrow 0$$

Curvatures

Curvatures $\quad t \to 0$:

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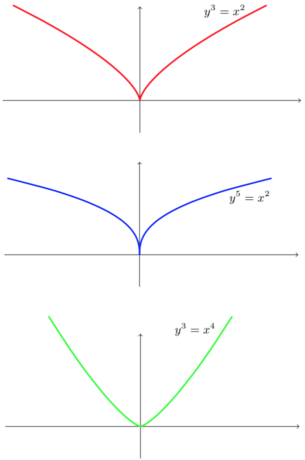
$$t \to (t^5, t^2) : \kappa = t^4/t^3 \longrightarrow 0$$

$$t \to (t^3, t^4) : \kappa = t^4/t^6 \longrightarrow \infty$$



Differentiability

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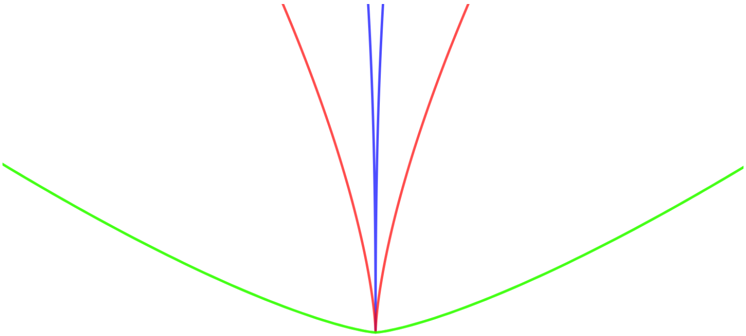
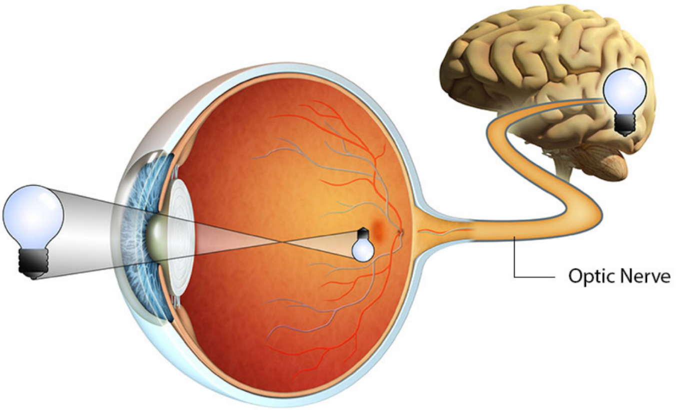
$\Rightarrow$ not C^2 at 0.



The Human Eye



How do we perceive geometric objects with our eye and our brain?

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Points – Lines – Curves – Directions – Locations.

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Euclid (approx. 300 BC):

'Ενα σημείο είναι αυτό δέν έχει κανένα μέρος.

How do we perceive geometric objects with our eye and our brain?

Points – Lines – Curves – Directions – Locations.

Euclid (approx. 300 BC):

'Ενα σημείο είναι αυτό δέν έχει κανένα μέρος.

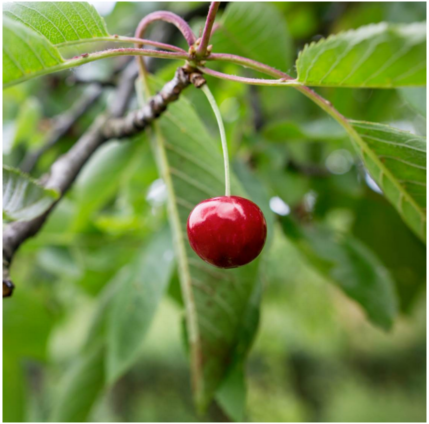
A point is that which has no part.

Not very helpful

to pick a cherry from a cherry tree

Not very helpful

to pick a cherry from a cherry tree



or to spot a mouse

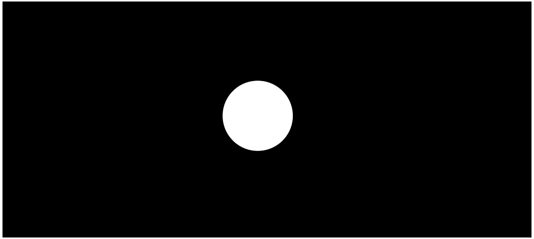
or to spot a mouse

Points

We start with a point

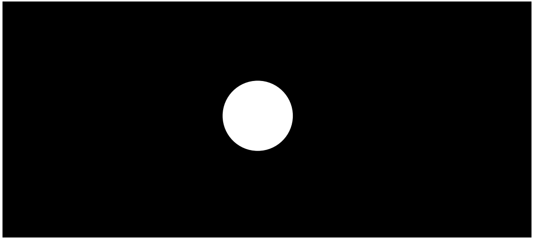
Points

We start with a point



Points

We start with a point



A point is the characteristic function of a small disk.

Points

Here is another point

Points

Here is another point

Are they different?

Some Physiology

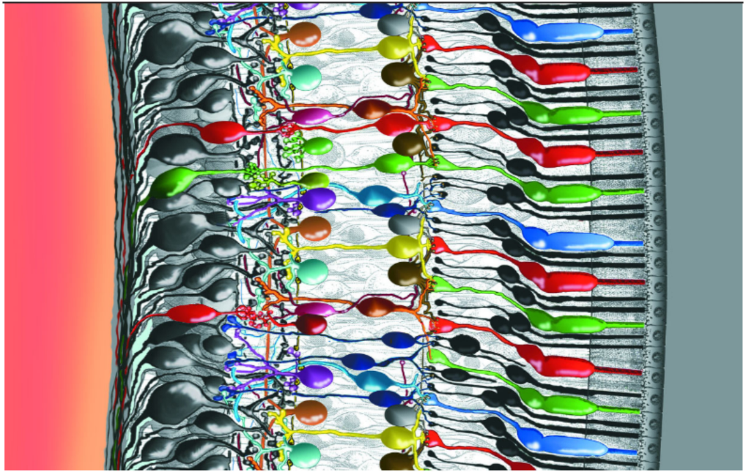
Retina – Cones – Rods – Photons – Ganglion Cells – Pathway Fibers – Optic Nerve

Some Physiology

Retina – Cones – Rods – Photons – Ganglion Cells – Pathway Fibers – Optic Nerve

0.5 mm thick
35 mm round
130.000.000 receptor cells
10.000.000 intermediate cells
1.000.000 ganglion cells
1.000.000 fibers

The Retina



(drawing courtesy of Helga Kolb)

Firing

Neuronal cells can only shoot

Firing

Neuronal cells can only shoot

Firing

Neuronal cells can only shoot

Once!

Firing

Reload and shoot again

$$\text{Intensity} = \text{Frequency!}$$

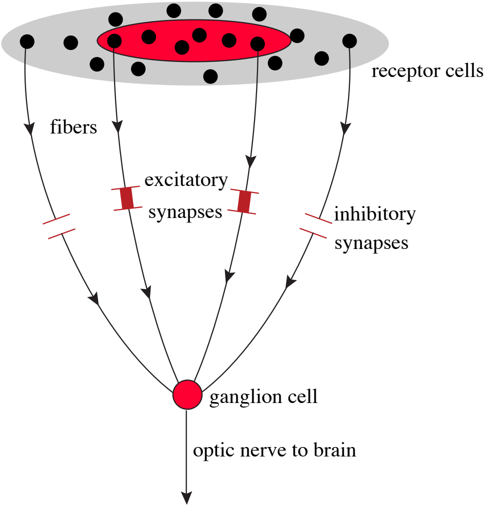
Two Principles

- Abundance (of bricks)
- Economy (of information)

Three Facts

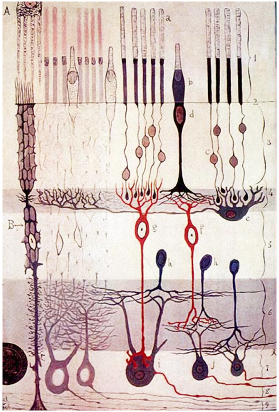
- brightness irrelevant
- contour and contrast count
- local geometry suffices

Receptive Field of Ganglion Cell:



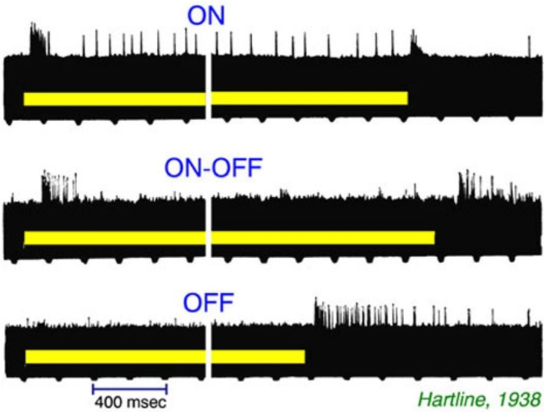
The Protagonists

Ramón y Cajal

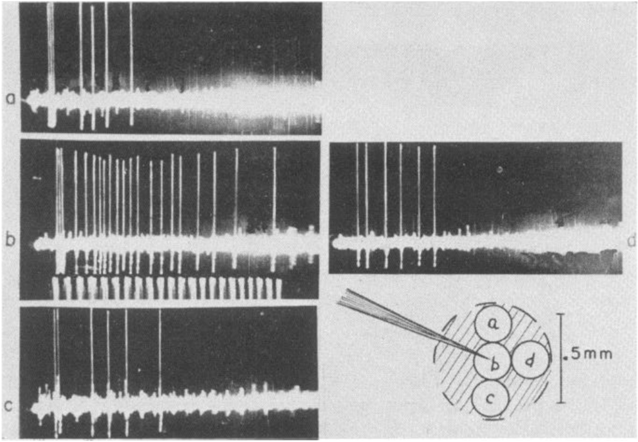


Ganglion Responses

Edgar Adrian & Keffer Hartline

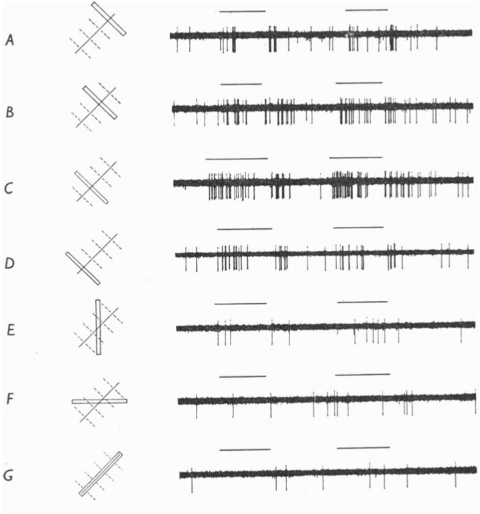


Stephen Kuffler



Ganglion Responses

David Hubel & Thorsten Wiesel



Retinal Pathways

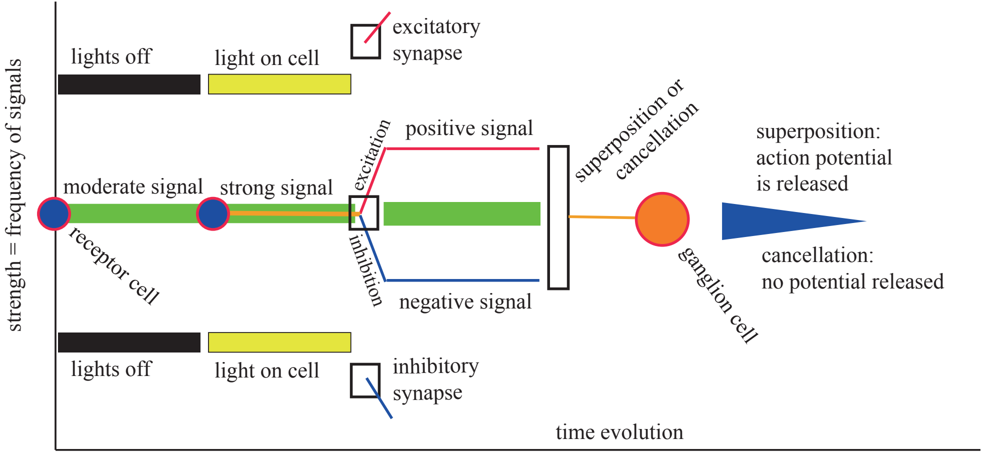
Retinal Pathways:

Light on Receptor Cell – Signal – Excitatory Synapse – Signal – Ganglion Cell – Action Potential

Light on Receptor Cell – Signal – Inhibitory Synapse – No Signal – Ganglion Cell – No Action Potential

Superposition: Excitatory $+$ Inhibitory $=$ Cancellation

Retinal Pathway:

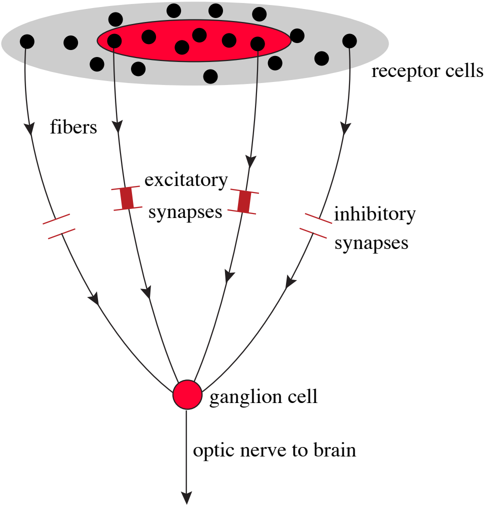


Off Response:

Inhibitory Synapse – Light off – Inhibition stopped – Positive Signal – Firing

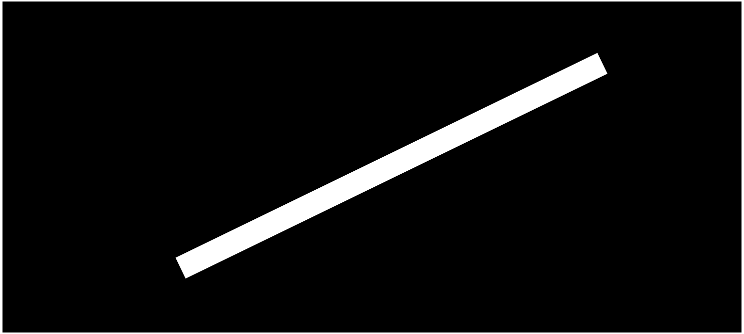
We see a point ...

Processing the picture of a bright point on dark background:



We see a line ...

Processing the picture of a bright line on dark background:



We see a direction … and a curve

Processing the experience of a directional movement:

delayed firing of receptor cell

We see a direction ... and a curve

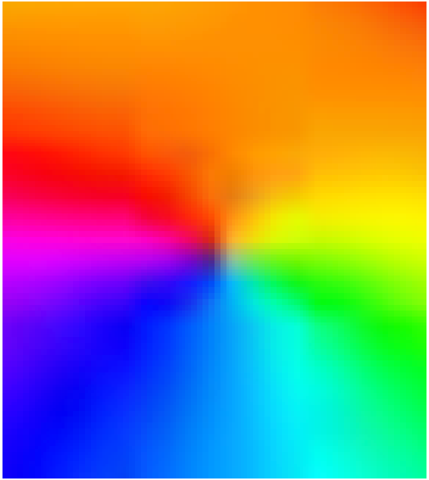
Processing the experience of a directional movement:

delayed firing of receptor cell

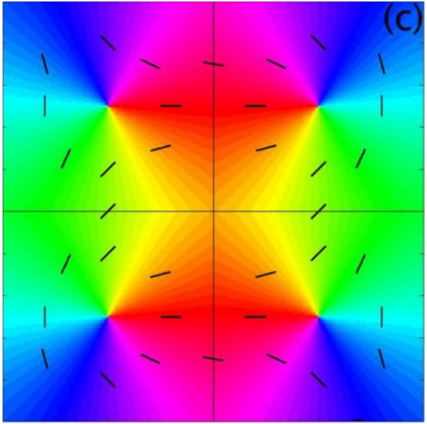
Processing the picture of a curve:

integration of vector fields

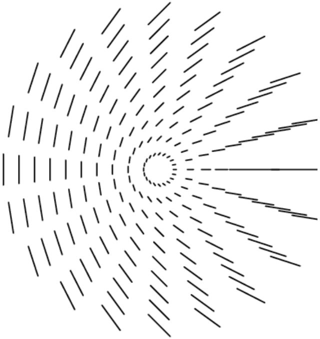
Directional sensitivity of cortical cells



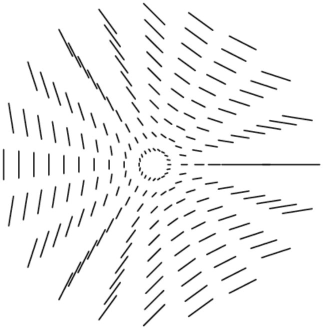
Directional sensitivity of cortical cells



Directional sensitivity of cortical cells



$z'(t) = z(t)^{1/2}$



$z'(t) = z(t)^{-1/2}$

The End ...