# *French-Chinese School on*

# *Differential and Functional Equations*

Wuhan University

Wuhan, China, April 16th-27th, 2012

# *French-Chinese School on Differential and Functional Equations*

## Wuhan, China, April 16th-27th, 2012

## Schedule

## The First Session: Mini-courses (April 16~20)

| | **Monday April 16** | **Tuesday April 17** | **Wednesday April 18** | **Thursday April 19** | **Friday April 20** |
|---|---|---|---|---|---|
| **08:00 – 09:40** | Michael Singer | Weinian Zhang | Guy Casale | Lucia Di Vizio | Ziming Li |
| **09:55-11:35** | David Blazquez-Sanz | Guy Casale | Lucia Di Vizio | Michael Singer | Charlotte Hardouin |
| **12:00-13:00** | Lunch | | | | |
| **14:00-15:40** | Weinian Zhang | Michael Singer | Ziming Li | David Blazquez-Sanz | |
| **15:55-17:35** | Guy Casale | David Blazquez-Sanz | Weinian Zhang | Ziming Li | |

**Place: Lecture Hall 205, School of Mathematics and Statistics.**

地点：数学院 **205** 报告厅

# The Second Session: Workshop (April 24~27)

| **Tuesday**<br>**April 24** | **Wednesday**<br>**April 25** | **Thursday**<br>**April 26** | **Friday**<br>**April 27** | |
|---|---|---|---|---|
| Opening(Photo)<br>(9:00-9:40) | | | | **Time**<br>**(April 25-27)** |
| J.-P. Ramis<br>(9:40-10:25) | Elie Compoint | Xiaoshan Gao | Chunming Yuan | 9:00-9:45 |
| Yongchuan Chen<br>(10:30-11:15) | Shuping Li | Laura Desideri | J.-A. Weil | 9:50-10:35 |
| | | | | Tea Break 10:40-11:05 |
| Hidetoshi Tahara<br>(11:20-12:05) | C. Mitschi | Stéphane Malek | Jianguo Si | 11:05-11:50 |
| Lunch (12:10-14:30) | Lunch | | | 11:55-14:00 |
| Ruyong Feng<br>(14:30-15:15) | Shaoyun Shi | Shengfu Deng | Jacques Sauloy | 14:00-14:45 |
| L. Zapponi<br>(15:20-16:05) | Florian Heiderich | Bing Xu | Zhenya Yan | 14:50-15:35 |
| | | | | Tea Break 15:40-16:05 |
| Tea Break (16:10-16:35) | Andrea Pulita | M. Wibmer | Thomas Dreyfus | 16:05-16:50 |
| Ruiming Zhang<br>(16:35-17:20) | Shaoshi Chen | Zhuangchu Luo | | 16:55-17:40 |
| | **Banquet (18:00)** | | | |

# *French-Chinese School on Differential and Functional Equations*

## Wuhan, China, April 16th-27th, 2012

## The Second Session: Workshop (April 24~27)

## Schedule

### Tuesday, April 24, 2012

| | |
|---|---|
| **Chairman** | ***Hua Chen*** |
| **9:00-9:40** | ***Opening*** |
| **9:40-9:50** | ***Take Picture*** |
| **9:50-10:00** | ***Tea break*** |
| **10:00-10:45** | ***J.-P. Ramis***<br>***Wild topology of irregular curves, iso-irregular deformations of linear O.D.E.and dynamics of Painlevé equations*** |
| **10:50 -11:35** | ***Yongchuan Chen***<br>***The convergence argument for rational solutions of difference equations*** |
| **11:40-12:25** | ***Hidetoshi Tahara***<br>***Maillet type theorem, convolution equations and multisummability of formal solutions*** |
| **12:30 -14:30** | **Lunch** |
| **Chairman** | ***Hidetoshi Tahara*** |
| **14:30-15:15** | ***Ruyong Feng***<br>***On the structure of compatible rational functions*** |
| **15:20-16:05** | ***Leonardo Zapponi***<br>***Lamé differential operators with finite monodromy and the supersingular curve in characteristic 2*** |
| **16:10-16:35** | ***Tea Break*** |
| **16:35-17:20** | ***Ruiming Zhang***<br>***On Fourier transforms and q-special functions*** |
| **17:35** | **Supper** |

## Wednesday, April 25，2012

| | |
|---|---|
| **Chairman** | ***Xiao-Shen Gao*** |
| **9:00-9:45** | ***Elie Compoint*** |
| | ***Characterization of reduced forms*** |
| **9:50-10:35** | ***Shuping Li*** |
| | ***Bifurcations of a discrete prey-predator model with Holling type II functional response*** |
| **10:40-11:05** | ***Tea break*** |
| **11:05-11:50** | ***Claude Mitschi*** |
| | ***Parameterized monodromy*** |
| **11:55-14:00** | **Lunch** |
| **Chairman** | ***Lucia Di Vizio*** |
| **14:00-14:45** | ***Shaoyun Shi*** |
| | ***Galoisian obstruction to the integrability of general dynamical systems*** |
| **14:50-15:35** | ***Florian Heiderich*** |
| | ***Galois theory of non-linear functional equations*** |
| **15:40-16:05** | ***Tea Break*** |
| **Chairman** | ***Ruyong Feng*** |
| **16:05-16:50** | ***Andrea Pulita*** |
| | ***Properties of the radius of convergence of p-adic differential and difference equations*** |
| **16:55-17:40** | ***Shaoshi Chen*** |
| | ***Telescopers for 3D walks via residues*** |
| **18:00** | **Banquet** |

## Thursday, April 26, 2012

| | |
|---|---|
| **Chairman** | ***J.-P. Ramis*** |
| **9:00-9:45** | ***Xiao-shan Gao*** |
| | ***Sparse differential resultant for Laurent differential polynomials*** |
| **9:50-10:35** | ***Laura Desideri*** |
| | ***Describing minimal surfaces using isomonodromic deformations*** |
| **10:40-11:05** | ***Tea break*** |
| **11:05-11:50** | ***Stéphane Malek*** |
| | ***On Gevrey solutions of threefold singular nonlinear PDEs*** |
| **11:55-14:00** | **Lunch** |
| **Chairman** | ***Wenyi Chen*** |
| **14:00-14:45** | ***Shengfu Deng*** |
| | ***Three-dimensional gravity-capillary waves on water — a spatial dynamical approach*** |
| **14:50-15:35** | ***Bing Xu*** |
| | ***On approximate and exact solutions of the linear functional equation of higher order*** |
| **15:40-16:05** | ***Tea Break*** |
| **Chairman** | ***Xiaochun Liu*** |
| **16:05-16:50** | ***Michael Wibmer*** |
| | ***Linear differential equations depending on a difference parameter*** |
| **16:55-17:40** | ***Zhuangchu Luo*** |
| | ***On the summability of the formal solution for some nonlinear singular PDEs*** |
| **17:55** | **Supper** |

## Friday, April 27，2012

| | |
|---|---|
| **Chairman** | ***Michael Singer*** |
| **9:00-9:45** | ***Chunming Yuan*** |
| | ***Differential chow form*** |
| **9:50-10:35** | ***Jacques-Arthur Weil*** |
| | ***Reduced forms of linear differential systems and applications*** |
| **10:40-11:05** | ***Tea break*** |
| **11:05-11:50** | ***Jianguo Si*** |
| | ***Construction of solutions for Feigenbaum's functional equations*** |
| **11:55-14:00** | **Lunch** |
| **Chairman** | ***Changgui Zhang*** |
| **14:00-14:45** | ***Jacques Sauloy*** |
| | ***Analytic classification and Galois theory for q-difference equations with two arbitrary slopes*** |
| **14:50-15:35** | ***Zhenya Yan*** |
| | ***Matter wave solutions in Bose-Einstein condensates*** |
| **15:40-16:05** | ***Tea Break*** |
| **16:05-16:50** | ***Thomas Dreyfus*** |
| | ***Kovacic's algorithm for parameterized linear differential equations of order 2*** |
| **16:55-17:40** | |
| **17:55** | **Supper** |

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# ABSTRACTS

# Telescopers for 3D Walks via Residues

Shaoshi Chen

Department of Mathematics, North Carolina State University,
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Telescopers are linear differential (recurrence) operators satisfied by definite integrals (sums) of certain class of functions (sequences). They are extensively used by combinatorists, such as Wilf, Zeilberger, Gessel etc. to enumerate combinatorial objects through generating functions and to show identities involving integrals or sums of special functions. In this talk, we present a criterion for deciding the existence of telescopers for rational functions which relates to combinatorial problems and an algorithm to construct them if they exist. Our approach is based on the investigation of residues of the input rational functions. We show that the problem of constructing telescopers for rational functions of three variables is equivalent to the problem of constructing telescopers for algebraic functions of two variables. With our method, we will answer some challenging problems on three-dimensional walks. This is a joint work with Manuel Kauers (RISC) and Michael F. Singer (NCSU).

# The Convergence Argument for Rational Solutions of Difference Equations

Yongchuan Chen

Center for Combinatorics, Nankai University, Tianjin 300071, China

# Characterization of Reduced Forms

Elie Compoint

Département de Mathématiques, Universitéde Lille 1,
Cité Scientifique, 59655 Villeneuve d'Ascq, France

Consider a linear differential system $[A] : Y' = AY$ with coefficients in a differential field $k$ (typically, $k = C(x)$). This system is said to be in reduced form when $A \in \mathfrak{g}(k)$, where $\mathfrak{g}$ denotes the Lie algebra of its differential Galois group. In this talk, we will give a characterization of reduced form in terms of Ivanriants and semi-invariants of the differential Galois group.

## Three-Dimensional Gravity-Capillary Waves on Water —A Spatial Dynamical Approach

Shengfu Deng
School of Mathematics and Computation Science,
Zhanjiang Normal University, Zhanjiang, Guangdong 524048, China

We consider three-dimensional gravity-capillary waves on water of finite-depth, which are uniformly translating in a horizontal propagating direction and periodic in a transverse direction. The exact Euler equations are formulated as a spatial dynamic system in which the variable used for the propagating direction is a time-like variable. A center-manifold reduction technique and a normal form analysis are applied to show that the dynamical system can be reduced to a system of ordinary differential equations. Using the existence of a homoclinic orbit connecting to a two-dimensional periodic (called generalized solitary-wave) solution for the reduced system, it is shown that such a generalized solitary-wave solution persists for the original system by applying a perturbation method and adjusting some appropriate constants.

## Describing minimal surfaces using isomonodromic deformations

Laura Desideri
Département de Mathématiques, Universitéde Lille 1,
Cité Scientifique, 59655 Villeneuve d'Ascq, France

I will present a correspondence due to R. Garnier between minimal surfaces with a polygonal boundary curve and a certain class of Fuchsian equations. In this correspondence, the monodromy of an equation is prescribed by the edge directions of the polygonal boundary curve of the associated minimal surface. We will see how isomonodromic deformations can then provide us with an explicit description of minimal disks, that can be used to solve the Plateau problem.

## Kovacic's Algorithm for Parameterized Linear Differential Equations of Order 2

Thomas Dreyfus
Institut de Mathématiques de Jussieu (IMJ), Université Pierre et Marie Curie,
4 place Jussieu, 75252 Paris Cedex, France

Let us consider a linear differential equation of the form $y''(X) = a(X)y(X)$, where $a(X)$ is a complex rational function. Kovacic's algorithm find the Liouvillian solutions and compute the differential Galois group. Recently, it has been developped a Galois theory for parameterized linear differential equations. We will see how to adapt the original Kovacic's algorithm in the parameterized case.

## On the Structure of Compatible Rational Functions

Ruyong Feng

Academy of Mathematics and Systems Science
No.55 Zhongguancun East Road, Haidian District, Beijing 100190, China

A finite number of rational functions are compatible if they satisfy the compatibility conditions of a first-order linear functional system involving differential, shift and q-shift operators. In this talk, we present a theorem that describes the structure of compatible rational functions. The theorem enables us to decompose a solution of such a system as a product of a rational function, several symbolic powers, a hyperexponential function, a hypergeometric term, and a q-hypergeometric term. Some applications are also presented in this talk. This is joint work with Shaoshi Chen, Guofeng Fu an Ziming Li.

## Sparse Differential Resultant for Laurent Differential Polynomials

Xiao-Shan Gao

Academy of Mathematics and Systems Science,
Chinese Academy of Sciences, Beijing 100190, China

In this talk, we will introduce the concept of Laurent differentially essential systems and give a criterion for Laurent differentially essential systems in terms of their supports. Then the sparse differential resultant for a Laurent differentially essential system is defined and its basic properties are reported. In particular, order and degree bounds for the sparse differential resultant are given. Based on these bounds, an algorithm to compute the sparse differential resultant is proposed, which is single exponential in terms of the order, the number of variables, and the size of the Laurent differential system.

## Galois theory of non-linear functional equations

Florian Heiderich

Institut de Mathématiques de Jussieu (IMJ), Université Pierre et Marie Curie,
4 place Jussieu, 75252 Paris Cedex, France

In Galois theory of differential and difference equations differential and difference rings play a central role, respectively. Both are a special cases of $D$-module algebras, where $D$ is a bialgebra. Recently, Moosa and Scanlon defined generalized Hasse-Schmidt rings, which are $D$-module algebras as well. For certain extensions of Artinian simple $D$-module algebras $L/K$, we define a group functor on the category of $L$-algebras. Its points are transformations fulfilling certain partial differential equations. In the case of Picard-Vessiot extensions this functor becomes isomorphic to the formal group scheme associated to the Galois group scheme after a finite étale

extension and under adequate conditions. This theory, aimed at the study of non-linear functional equations unifies and generalizes the Galois Theory for non-linear differential equations due to Umemura and its difference analogue due to Morikawa-Umemura.

## Bifurcations of a discrete prey-predator model with Holling type II functional response

Shuping Li

School of Mathematical Sciences, University of Science and Technology of China,
No.96, JinZhai Road Baohe District,Hefei,Anhui, 230026, P.R.China

We apply center manifold reduction and the method of normal forms to completely discuss bifurcations of a discrete prey-predator model. We give bifurcation curves analytically for transcritical bifurcation, flip bifurcation， Neimark-Sacker bifurcation and strong resonance separately, showing bifurcation phenomena not indicated in the previous work for the system.

## On the Summability of the Formal Solution for Some Nonlinear Singular PDEs

Zhuangchu Luo

School of Mathematics and Statistics, Wuhan University, Wuhan 430072, China

In this paper, we discuss a class of first order nonlinear degenerate partial differential equations with singularity at $(t, x) = (0, 0) \in \mathbf{C}^2$. Using exponential-type Nagumo norm approach, the Gevrey asymptotic analysis is extended to the case of holomorphic parameters in a natural way. A sharp condition is then established to deduce the $k$-summability of the formal solutions. Furthermore, analytical solutions in conical domains are found for each type of these nonlinear singular PDEs.

This talk is based on a joint work with Hua Chen and Changgui Zhang.

## On Gevrey solutions of threefold singular nonlinear PDEs

Stéphane Malek

Département de Mathématiques, Universitéde Lille 1,
Cité Scientifique, 59655 Villeneuve d'Ascq, France

We study Gevrey asymptotics of the solutions to a family of threefold singular nonlinear partial differential equations in the complex domain. We deal with both Fuchsian and irregular singularities, and allow the presence of a singular perturbation parameter. By means of the Borel-Laplace summation method, we construct sectorial actual holomorphic solutions which

turn out to share a same formal power series as their Gevrey asymptotic expansion in the perturbation parameter.

This result rests on the Malgrange-Sibuya theorem, and it requires to prove that the difference between two neighboring solutions is exponentially small, what in this case involves an asymptotic estimate for a particular Dirichlet-like series.

## Parameterized monodromy

Claude Mitschi

Institut de Recherche Mathématique Avancée, CNRS - Université de Strasbourg,
7 rue R. Descartes, 67084 Strasbourg, France

In joint work with Michael F. Singer, we have studied analytic families of differential systems with parameterized regular singularities and given different properties of their parameterized monodromy. Our results include a parameterized version of the Schlesinger theorem and of the Riemann-Hilbert correspondence, which naturally led to an analogue in parameterized Picard-Vessiot theory of Tretkoff's solution of the global inverse problem in (constant) differential Galois theory over the rational functions.

## Properties of the radius of convergence of $p$-adic differential and difference equations

Andrea Pulita

Département de Mathématiques Universitéde Montpellier II, CC051
Place Eugène Bataillon, F-34 095 Montpellier CEDEX 5, France

We define the radius of convergence function of a $p$-adic differential or difference equation. This function is defined on a certain Berkovich space and it encodes a lot of numerical invariants of the equation. We give then the main properties of this function, as for example the fact that it is a continuous function. Then we provide a proof of the fact the Radius of convergence function is determined by a finite sub-tree of the Berkovich space. This have been conjectured by F. Baldassarri in 2010.

# Wild topology of irregular curves, iso-irregular deformations of linear O.D.E. and dynamics of Painlevé equations

Jean-Pierre Ramis

Institut Mathématiques de Toulouse (Laboratoire Emile Picard),
Université Paul Sabatier, 118, route de Narbonne, 31062 Toulouse CEDEX 9,France

We will present a "work in progress" in collaboration with Julio Rebelo. It is mainly a *program*, with some conjectural parts.

Our main purpose is to "understand the dynamics" of the six Painlevé equations, in particular to be able *to compute their non-linear differential Galois groupoids in Malgrange sense*. We conjecture that, for "generic values" of the parameters, these groupoids are "as big as possible": *conservation of the area* (as for $P_I$, a result of G. Casale).

The results are known for Painlev?é six (Cantat, Loray, Iwasaki, M.H. Saito...) and our idea is to "imitate" the method for $P_V,\ P_{IV},\ P_{III},\ P_{II},\ P_I$.

In the case of $P_{VI}$ one *translates*, via the Riemann-Hilbert correspondence, the *transcendental* initial problem (the study of the dynamics of the equation, or equivalently of the study of the *nonlinear* monodromy around $0, 1, \infty$) into a *purely topological problem*: the study of the dynamics induced by a *braid group on a character variety*. The character variety is an *affine* algebraic *surface* (with a complex symplectic structure) and the action is *polynomial and explicit*.

In the case of $P_V,\ P_{IV},\ P_{III},\ P_{II},\ P_I$, we can *translate*, via the *irregular* Riemann-Hilbert correspondance (in Martinet-Ramis style), the *transcendental* initial problem into a new one. Now the new problem is *no longer* purely topological. It is necesary to replace the four punctured sphere by some *irregular curves*, the braid group by some *wild braid groups* and the character varieties by some *wild character varieties* (replacing representations of a fundamental group by representations of a *wild fundamental groupoid*).

We will explain the mechanism in a quite general situation and afterwards we will describe what is happening in the case of the irregular Painlevé equations, using some recent results of M. van der Put and M.H. Saito.

# Analytic classification and Galois theory for q-difference equations with two arbitrary slopes

Jacques Sauloy

Institut Mathématiques de Toulouse (Laboratoire Emile Picard),
Université Paul Sabatier, 118, route de Narbonne, 31062 Toulouse CEDEX 9,France

Analytic classification and Galois theory for q-difference equations with integral slopes was obtained by Ramis, Sauloy and Zhang in the last decade. In her thesis, Virginie Bugeaud started the extension of these results to arbitrary slopes. Up to now, only the "abelian" case of two slopes is understood.

# Galoisian obstruction to the integrability of general dynamical systems

Shaoyun Shi

College of Mathematics, Jilin University,
No.2699 Qianwei Road Changchun,Jilin, P.R.China

The Galoisian approach to study the integrability of classical Hamiltonian systems, the so-called Morales-Ramis theory, has been proved to be useful and powerful by many applications. Here, two analogous forms of the Morales-Ramis theory for general dynamical systems both in vector field and mapping forms are given. Galois groups of the corresponding variational equations are studied, and some necessary conditions of the system to possess a certain number of integrals are presented.

# Construction of solutions for Feigenbaum's functional equations

Jianguo Si

School of Mathematics , Shandong University,
Jinan, Shandong 250100, People's Republic of China

In this talk, we consider the existence of convex solutions for Feigenbaum's functional equations:

$$\begin{cases} f(x) = \frac{1}{\lambda} f(f(\lambda x)), \quad 0 < \lambda < 1, \\ \\ f(0) = 1, \quad 0 \le f(x) \le 1, \quad x \in [0,1]. \end{cases}$$

# Maillet type theorem, convolution equations and multisummability of formal solutions

Hidetoshi Tahara

Department of Mathematics, Sophia University, Tokyo, Japan

In this talk, I will consider the following linear singular partial differential equation:

$$\text{(E)} \qquad P(t\partial_t)u = \sum_{j+|\alpha|\le L} a_{j,\alpha}(t)(t\partial_t)^j \partial_x^\alpha u + f(t,x)$$

with $(t,x) \in \mathbb{C}_t \times \mathbb{R}_x^N$ (or $(t,x) \in \mathbb{C}_t \times \mathbb{C}_x^N$) with holomorphic coefficients $a_{j,\alpha}(t)$. First, I will present a Maillet type theorem for formal solutions of this equation (E), then I will give an analogue of Maillet type theorem in convolution partial differential equations, and finally I will give an application to multisummability of formal solutions of (E).

## Reduced Forms of Linear Differential Systems and applications

Jacques-Arthur Weil

Département de Mathématiques Informatique, XLIM - UMR CNRS 6172,
Université de Limoges, 123 avenue Albert Thomas, 87060 Limoges cedex, France

Consider a linear differential system $[A]: Y' = AY$ with coefficients in a differential field $k$ (typically, $k = C(x)$). This system is said to be in reduced form when $A \in \mathfrak{g}(k)$, where $\mathfrak{g}$ denotes the Lie algebra of its differential Galois group. In this talk, we will review useful aspects of reduced forms, and address constructive aspects on how to transform a system into a reduced form.

## Linear differential equations depending on a difference parameter

Michael Wibmer

Lehrstuhl A für Mathematik, RWTH Aachen,52056 Aachen, Germany

This is joint work with Lucia Di Vizio and Charlotte Hardouin. A Galois theory for linear differential and difference equations whose coefficients depend on differential parameters has been developed by Cassidy, Hardouin, Singer and others. In this talk we will present a Galois theory for linear differential equations depending on a difference parameter. This theory is designed to study the difference algebraic relations among the solutions of a linear differential equation. The Galois groups are linear difference algebraic groups, i.e., matrix groups defined by difference equations. Along the way we shall encounter some basic theorems from difference algebra.

## On approximate and exact solutions of the linear functional equation of higher order

Bing Xu

Department of Mathematics, Sichuan University, Chengdu, Sichuan 610064, P.R. China

Joint work with Janusz Brzdęk, Dorian Popa.

We show that, under some assumptions, every approximate solution of the linear functional equation

$$\varphi(f^m(x)) = \sum_{i=1}^{m} a_i(x)\varphi(f^{m-i}(x)) + F(x)$$

generates a solution of the equation that is close to it. We also give a description of a procedure that yields such a solution, estimate the distance between those approximate and exact solutions to the equation, and discuss the problem of uniqueness. Moreover, as a consequence we obtain some results concerning the Hyers-Ulam stability of the equation.

## Matter wave solutions in Bose-Einstein condensates

Zhenya Yan

Key Laboratory of Mathematics Mechanization, Institute of Systems Science, AMSS, Chinese Academy of Sciences, Beijing 100190, China

The Gross-Pitaevskii (GP) equation is used to describe nonlinear wave phenomenon in Bose-Einstein condensates, which is also called nonlinear Schrödinger equation in nonlinear optics. In this talk, we firstly introduction the recent development on exact solutions of the Gross-Pitaevskii equation and its extensions. Secondly, we focus on the studies of the solutions for 1D GP equation with the van der Waals potential on a semi-line $x > 0$, 2D GP equation with some potentials in a domain $D \in \mathbb{R}^2$ (i.e., the Dirichlet problem), the 3D generalized GP equation, the 1D GP equation with a source. Finally, we given some conclusions and discussions.

## Differential Chow Form

Chunming Yuan

Key Lab of Mathematics Mechanization, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, No.55 Zhongguancun East Road, 100190, Beijing, China

In this talk, an intersection theory for generic differential polynomials is presented. The intersection of an irreducible differential variety of dimension $d$ and order $h$ with a generic differential hypersurface of order $s$ is shown to be an irreducible variety of dimension $d-1$ and order $h+s$. As a consequence, the dimension conjecture for generic differential polynomials is proved. Based on the intersection theory, the Chow form for an irreducible differential variety is defined and most of the properties of the Chow form in the algebraic case are extended to its differential counterpart. Furthermore, the generalized differential Chow form is defined and its properties are proved. As an application of the generalized differential Chow form, the differential resultant of $n+1$ generic differential polynomials in $n$ variables is defined and properties similar to that of the Sylvester resultant of two univariate polynomials are proved.

## Lamé differential operators with finite monodromy and the supersingular curve in characteristic 2

Leonardo Zapponi

Institut de Mathématiques de Jussieu, Université Pierre et Marie Curie, 4 place Jussieu, 75252 Paris Cedex, France

Following a criterion of Baldassarri, the existence of Lamé operators with finite monodromy is related to the existence of particular torsion points on elliptic curves. These points can be defined in positive characteristic and have a particular nice behaviour under specialization. Using this approach, we show that Lamé operators with finite monodromy are naturally parametrized by the points of the supersingular elliptic curve in characteristic 2.

# On Fourier Transforms and $q$-Special Functions

Ruiming Zhang

The Institute of Applied Mathematics
College of Science, Northwest A&F University
Yangling, Shaanxi 712100, P. R. China.

This talk is based on a joint work in progress with professor Mourad E. H. Ismail. In this talk we show how to prove certain identities and asymptotic behaviors for some $q$-special functions via Fourier transformations.

# Participants

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