Arithmetic study of linear q-difference equations

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Wuhan, 26-17/04/2012

The aim of this course is to show how one can my describe the algebraic relations satisfied by the solutions of a q-difference equation via the algebraic relations satisfied by the curvatures of the same equation. It will be articulated in the following sections:

1. Introduction to global fields: Dedekind ring and valuations; extension of valuations; global fields with the product formula.

2. Introduction to Tannakian categories. The aim of the formalism of Tannakian category is to answer the question: "When is a category equivalent to the category of representations of a an affine group scheme G? How can we recover G from such a category?". We will explain the main concepts of the theory and the

3. Grothendieck conjecture for linear q-difference equation: Dwork's rationality criteria; curvatures of a q-difference equation; generic Galois groups; characterization *a la Katz* of the generic Galois groups of a q-difference equation.

4. We will end by explaining the links with M. Singer's course on the differential properties of solutions of linear difference equations.