Galois Theory of Linear Difference Equations: An algebraic and Algorithmic Perspective

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This short course will cover the following topics:

1. What is a linear difference equation and what is a solution? I will discuss the equivalence of scalar equations, linear systems and difference modules and the differences in thinking of solutions as analytic functions, sequences or formal objects.

2. Basic Galois Theory and Applications. I will give an overview of the algebraic approach to Galois theory and apply it to answer questions such as: How can one discover the relation $F(n+2)F(n) - F(n+1)2 = (-1)^n$ among the Fibonacci numbers $F(n)$? and: What sequences satisfy both a linear difference equation and a polynomial equation?

3. Algorithms. I will show how to answer the questions: How does one solve linear difference equations in "closed form"? How can one verify combinatorial identities?

4. Differential properties of solutions of linear difference equations- A classical result of Hoelder states that the Gamma function (which satisfies y(x+1) - x y(x) = 0) satisfies no polynomial differential equation with complex coefficients. I will show how an extension of the above Galois theory can be used to show this and how it can be used to characterize the possible differential relations among solutions of classes of linear differential equations.

I will have exercises and indications of directions of future research.